HYBRID MODE ANALYSIS OF MICROSTRIP LINES ON ANISOTROPIC SUBSTRATES

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ABSTRACT

A rigorous hybrid mode analysis is applied to the microstrip line on anisotropic substrate to determine its high frequency performance. The treatment is limited to substrates with uniaxial anisotropy with the principal axis perpindicular to the surface. This includes the practically important case of microstrip lines on Sapphire. The exact results obtained are used to check the validity of the previously introduced concept of equivalent isotropic substrate,which was used by some authors to simplify the calculations of the high frequency parameters of such lines .

I-Introduction

Crystalline substrates,e.g. Sapphire,are used in microstrip lines intended for use in some applications.They have certain advantages over ceramics,which include:lower losses, higher homogeniety and lower variations of electrical parameters from specimen to specimen.However,being anisotropic,they lead to electrical performance of lines which differs somewhat from that of lines on isotropic substrates usually used .

The quasi-static characteristics of microstrip lines on anisotropic substrates have been investigated by several authors /1-5/. They studied mainly the static parameters of the lines,obtained from the solution of electrostatic problems .Some authors suggested that the effect ofanisotropy can be accounted for by the introduction of some equivalent isotropic medium with some effective parameters.Thus,with uniaxially anisotropic substrates cut with the optical axis perpindicular to the plane of the substrate,it was assumed /4/,that microstrip lines with substrate thickness d and relative dielectric constants in directions parallel and perpindicular to the axis ε_z, ε_t respectively,will behave as if they were isotropic with relative dielectric constant ε_e and effective substrate thickness d_e given by:

$$d_e = d\sqrt{\varepsilon_t/\varepsilon_z} \quad , \quad \varepsilon_e = \sqrt{\varepsilon_t\ \varepsilon_z} \quad .$$

Width-dependent equivalent parameters were introduced by Edwards & Owens/1,2/ and were shown to give sufficiently accurate dispersion characteristics when used with existing wave theories of microstrip lines on isotropic substrates .

The purpose of the present paper is to present a rigorous treatment of microstrip lines on anisotropic substrates and to check the validity of the previously introduced assumptions over an extended range of frequencies.

II-Formulation and Solution of the Problem:

The configuration of the microstrip line is shown on fig.1,where the coordinate axes are chosen with the z-axis perpindicular to the plane of the substrate.The permittivity tensor of the substrate material is taken in the form:

$$\hat{\varepsilon} = \varepsilon_o \begin{pmatrix} \varepsilon_t & 0 & 0 \\ 0 & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$$

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The analysis is performed using the function theoretic approach, which was applied to microstrips/6/. It can be shown, that the problem reduces to the solution of the two sets of equations :

$$A_n = 1 + \sum_{m=0}^{\infty} \frac{\xi_m}{\alpha_n + \alpha_m} A_m \ , \ n=0,1,2,..$$

$$B_n = 1 + \sum_{m=1}^{\infty} \frac{\zeta_m}{\alpha'_n + \alpha'_m} B_m \ , \ n=1,2,...$$

with coefficients

$$\xi_n = \frac{\operatorname{Res} \mathcal{X}_1^-(\alpha_n)}{\mathcal{X}_1^-(-\alpha_n)} e^{i\alpha_n W}$$

$$-\zeta_n = \frac{\operatorname{Res} \mathcal{X}_2^-(\alpha'_n)}{\mathcal{X}_2^-(-\alpha'_n)} e^{i\alpha'_n W}$$

where $\mathcal{X}_1^-(\alpha), \mathcal{X}_2^-(\alpha)$ are the minus-functions, resulting from the factorization of the functions:

$$\mathcal{X}_1(\alpha) = \frac{\operatorname{Coth} R_o d_o}{R_o} + \varepsilon_e \frac{\operatorname{Coth} R_z d_e}{R_z}$$

$$\mathcal{X}_2(\alpha) = R_o \operatorname{Coth} R_o d_o + \frac{1}{\mu_r} R_t \operatorname{Coth} R_t d \ ,$$

Where

$$R_o = \sqrt{\alpha^2 + \gamma^2 - k_o^2} \ , \ R_z = \sqrt{\alpha^2 + \gamma^2 - k_o^2 \mu_r \varepsilon_z} \ ,$$

$$R_t = \sqrt{\alpha^2 + \gamma^2 - k_o^2 \mu_r \varepsilon_t} \ , \ k_o^2 = \omega^2 \varepsilon_o \mu_o \ .$$

The dependence on the longitudinal coordinate y and the time t is taken in the form $e^{-i(\omega t - \gamma y)}$. α -is a complex variable used as a parameter in the Fourier transforms of the fields in the direction of x . α_n, α'_n are the poles of $\mathcal{X}_1, \mathcal{X}_2$ respectively in the upper half-plane of α .

Coefficients A_n, B_n define two functions

$$F_1^-(\alpha) = \frac{P}{\mathcal{X}_1^-} \sum_{n=0} \frac{\xi_n}{\alpha - \alpha_n} A_n \ ,$$

$$F_2^-(\alpha) = \frac{Q}{\mathcal{X}_2^-} \sum_{n=1} \frac{\zeta_n}{\alpha - \alpha'_n} B_n \ ,$$

which are some combinations of the transforms of the tangential components of the electric field at the plane of the strip $z=0$. The propagation constant γ is calculated from the conjugate conditions:

$$F_1^-(\pm i \gamma) \pm i F_2^-(\pm i \gamma) = 0 \ .$$

III-Numerical Results and Conclusions:

Comparing the expressions for $\mathcal{X}_1, \mathcal{X}_2$ with thier expressions in the isotropic case, i.e. when $\varepsilon_z = \varepsilon_t = \varepsilon_r$, we notice that, except for the root R_z, function $\mathcal{X}_1$ keeps its form. It was shown/6/, that the TM-to-z component of the field, which is described by the function $\mathcal{X}_1$, dominates at low frequencies. This leads to the conclusion, that the equivalent isotropic parameters can be introduced in the manner suggested by Horno/4/ and are expected to give accurate results in the quasi-static limit .

However, it can be shown , that due to the presence of the root R_z in $\mathcal{X}_1$ the high frequency limit of the effective microstrip dielectric constant ε_{eff} is always ε_z irrespective of the line dimensions and it approaches this limit faster as the strip width increases. This situation favours the use of width-dependent parameters if the microstrip dispersion is to be taken into account .

Computations have been performed using the present theory to calculate the high frequency performance of microstrip lines on Sapphire substrates with $\varepsilon_t=9.4$, $\varepsilon_z=11.6$ and dimensions of some of the lines investigated by Edwards & Owens/1,2/, and the results are given on fig.2-a,2-b, showing very good agreement with experimental measurements .

Calculations were also conducted using the equivalent isotropic parameters suggested by the same authors. They were compared withthe results obtained from the present theory, leading to the following conclusions:

1-Values of ε_{eff} and the line impedance Z_o calculated using the isotropic parameters suggested by Horno differ by about 4% at 5 GHz up to 9% at 40 GHz from the results of the theory. Therefore they can be used mostly for rough calculations .

2-The width-dependent equivalent isotropic substrate dielectric constant introduced by Edwards &Owens leads to values of ε_{eff} ,Z_o agreeing very well with the theory up to about 25 GHz and differ by less than 2% up to 40 GHz over the range of frequencies and geometries considered. Therefore this equivalent dielectric constant is expected to give accurate results over wide frequency range when used with simplified theory or model accounting for dispersion in microstrip with isotropic substrates for CAD .

3-When the microstrip is to be used at higher frequencies or when higher accuracy is required,rigorous theory has to be used for the determination of its performance.

References:

/1/R.P. Owens,J.E.Aitken and T.C. Edwards, Quasi-static characteristics of microstrip on an anisotropic Sapphire substrate,IEEE Trans.on Microwave Theory & Tech.MTT-24, August 1976.

/2/T.C. Edwards and R.P. Owens,2-18 GHz Dispersion measurements on 10-100 Ω micro-strip lines on Sapphire,IEEE Trans.Microwave Theory&Tech.MTT-24,August 1976.

/3/N.G. Alexopulos and C.M. Krowne,Characteristics of single and coupled microstrips on anisotropic substrates,IEEE Trans.Microwave theory&Tech.MTT-26,June 1978.

/4/M. Horno,Calculation of quasistatic characteristics of microstrip on anisotropic substrate using mapping method,IEEE MTT-S Int. Microwave Symp.,Washington 1980,p.450

/5/M.El-Said and A.A Ahmed,Microstrip analysis on anisotropic and/or inhomogeneous substrate with finite element method,IEEE MTT-S Int. Microwave Symp.,Washington 1980, p.465.

/6/A-M. A. El-Sherbiny,Exact analysis of shielded microstrip lines and bilateral finlines,IEEE MTT-S Int. Microwave Symp., Washington 1980,p.459 .

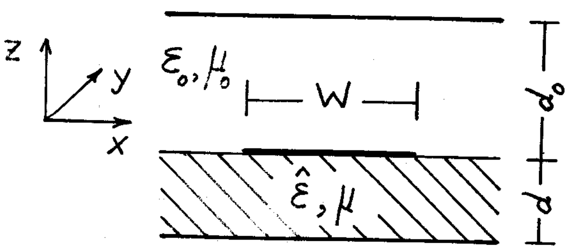

Fig. 1.- Microstrip configuration.

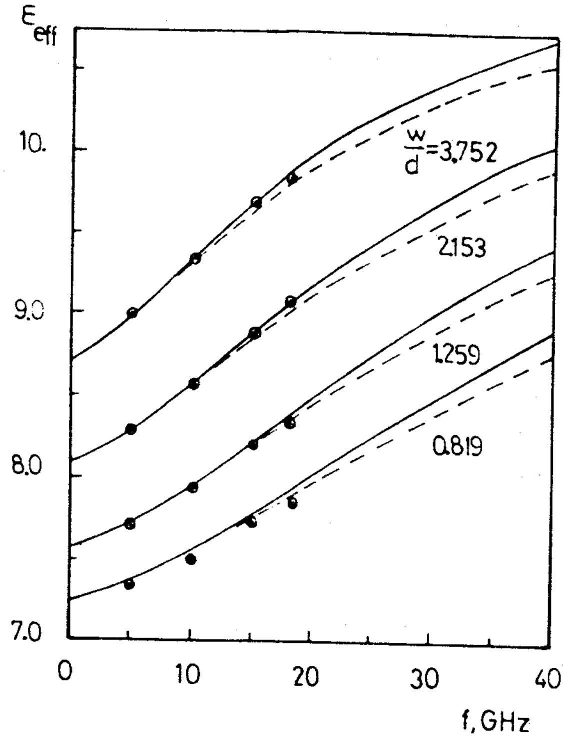

Fig.2-a : Dispersion characteristics of microstrip line on Sapphire substrate. ε_t=9.4 , ε_z=11.6 . —— theory, ----- results for equivalent isotropic substrate , ⊛ experimental results of /2/.

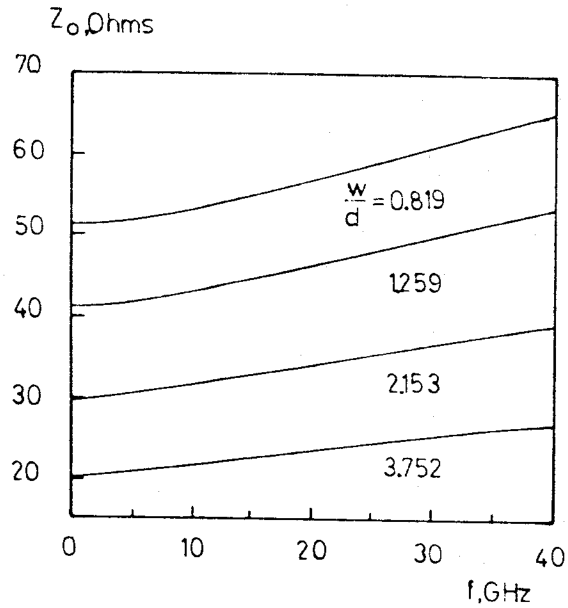

Fig.2-b:Characteristic impedance of microstrip line on Sapphire substrate as a function of frequency .